

This is a draft; please send me corrections and/or suggestions.

Gambling

You may have heard of the following gambling strategy, called the *Martingale strategy*: if you win a particular gamble, you collect your earnings; if you lose, you double the bet you just made and try again.

In a more formal setup, assume you are betting on the outcome of a fair coin, which lands heads-up with probability $\frac{1}{2}$ and tails-up with probability $\frac{1}{2}$. You first bet on this coin is \$2; if you win, you are done. But if you lose, you double your bet (to \$4) and try again. If you lose that bet, you double your bet again (to \$8) and try again. You repeat this process until you win a coin toss.

Let X be a random variable representing when you finally win the coin toss.

- (a) *What is the PMF of X ?*
- (b) *How much money have you won if $X = T$?*
- (c) *What is your expected profit?*

One unrealistic feature of this setup is that you can continue doubling your bet if you lose. Imagine you have lost 10 flips in a row: you will have to place an extremely large bet on the 11th flip if you hope to recoup your lost earlier investments.¹ So now, let's assume that you start gambling with only \$100, and you cannot gamble away money you don't have (so if you have m left on your bankroll and your next bet would be $b > m$, you have to just walk away).

- (d) *If you start with \$100, how many bets can you lose before you have to quit using the Martingale strategy?*
- (e) *Let Y be a random variable representing the time at which you finally win or walk away when your budget is constrained. What is the PMF of Y ?*
- (f) *What is your expected profit?*
- (g) *Thought experiment: what assumption have we made here that, while making the math friendly, is extremely unrealistic? If we change this assumption to make it better-match the real world, what happens to your expected profit in the previous question? Is the Martingale strategy smart? Bonus: can the strategy be adjusted to account for this (hint: think about what you could do if you made the math more difficult than, "double my previous bet.")?*

Investments

Here's a simple model of the stock market which actually follows some models that economists use.² There are two stocks: one can increase by 20% per year with probability $\frac{1}{2}$ or decrease by 10% per year with probability $\frac{1}{2}$; the other can increase by 60% per year with probability $\frac{1}{3}$ or decrease by 20% per year with probability $\frac{2}{3}$. You are concerned with investing over a period of two years; asset performance is independent between the first year and the second.

¹Depending on what you consider a "large" bet. On a graduate student salary, this bet is a dealbreaker.

²The formal name for this, if we consider a slightly broader class of investments, is the *binomial options pricing model*.

- (a) For each stock, what are the possible outcomes (increases or decreases in value) over two years?
- (b) Let X_i denote the relative return of stock i over two years. What is the PMF of X_1 ? X_2 ?
- (c) What is $\mathbb{E}[X_1]$? $\mathbb{E}[X_2]$?
- (d) Based solely on expected return, which stock would you rather purchase?
- (e) What is $\text{Var}(X_1)$? $\text{Var}(X_2)$?
- (f) Does knowing the variance change your decision regarding which stock you would rather purchase? Why or why not?

Class quizzes

The week-3 quiz has 10 questions. A student has a 90% chance of getting any one question right, independent of all others. Let X represent the number of correct answers on the quiz (the grade).

- (a) What is the distribution of X ?
- (b) What is $P(\text{all answers are correct})$? $P(\text{only one answer is wrong})$?
- (c) Is $\{\text{all answers are correct}\}$ independent of $\{\text{only one answer is wrong}\}$?
- (d) A friend tells you she missed **no more than one** question on the quiz. What is the probability that she achieved a perfect score?
- (e) Suppose that the TAs messed up, and two of the questions are identical; missing one of these questions means missing both. If X is the grade on the quiz, what is the PMF of X ?